

# Recurrent neural networks and hierarchical structure

Dieuwke Hupkes

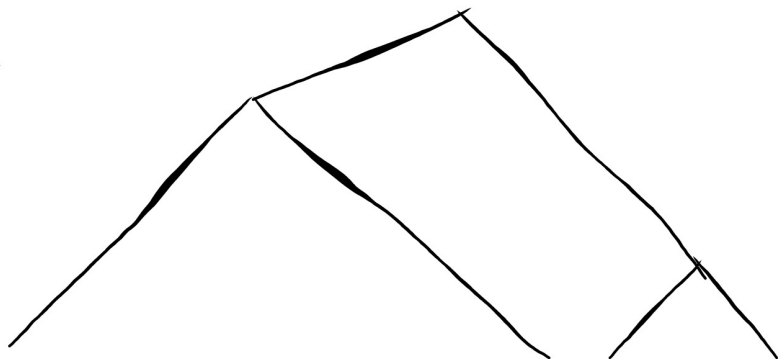
Institute for Logic, Language and Computation  
University of Amsterdam

Johns Hopkins University  
October 9, 2019

# The structure of language

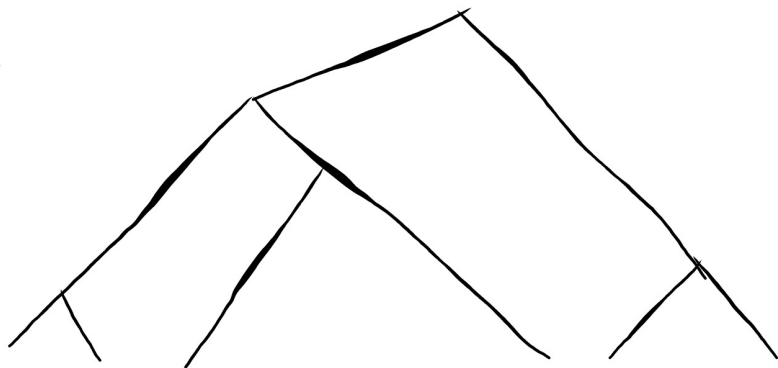
The scientist who wrote the research paper jumped with joy

# The structure of language



The scientist who wrote the research paper jumped with joy

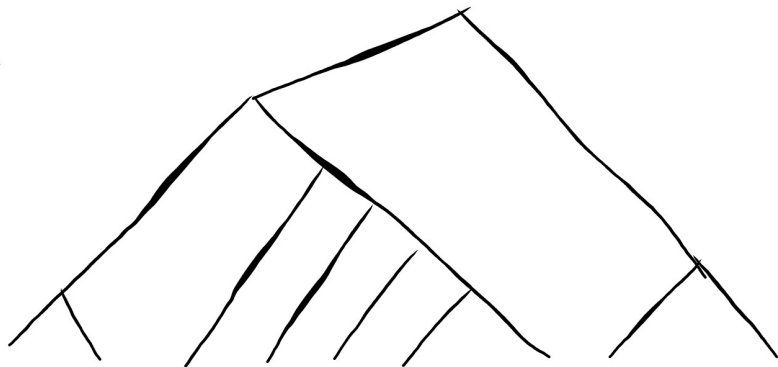
# The structure of language



The scientist who wrote the research paper jumped with joy

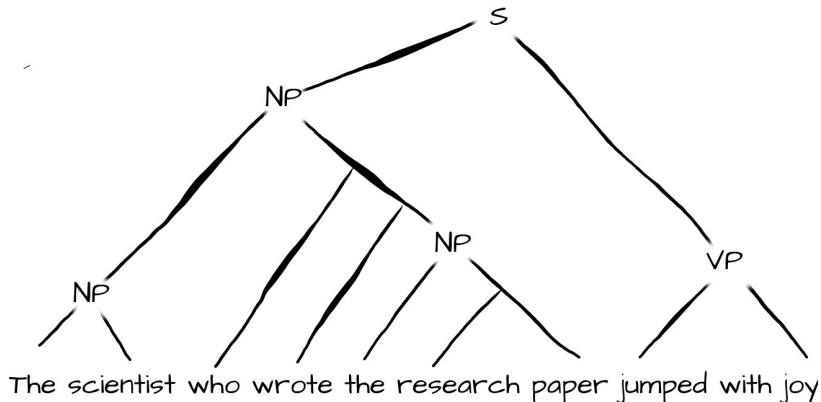


# The structure of language



The scientist who wrote the research paper jumped with joy

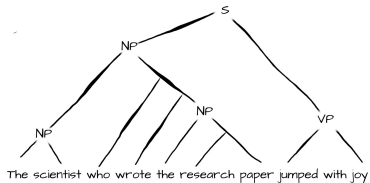
# The structure of language



# Symbolic structure and the brain



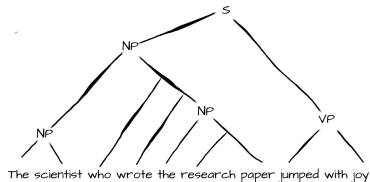
?



# Symbolic structure and the brain

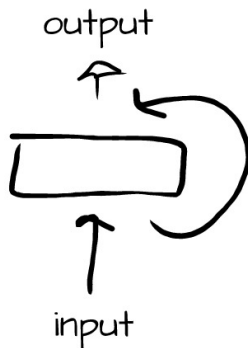


?



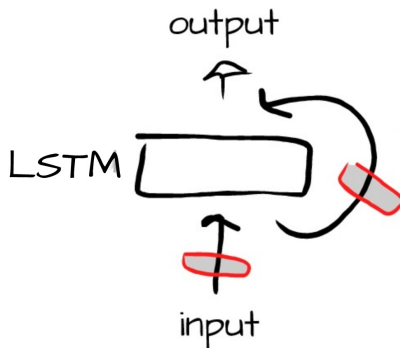
- Language is a product of our brain, but our brains do not have any explicit means to represent rules and symbols, how is this possible?

## Simple recurrent network



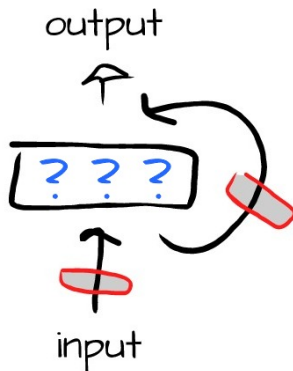
(Elman 1990)

# Gated recurrent neural networks



(Hochreiter and Schmidhuber 1997)

# Gated recurrent neural networks



- How can hierarchical structure be processed *incrementally*, in *linear time*, by a *recurrent artificial neural network*?

# Artificial languages

- The compositionality of neural networks: integrating symbolism and connectionism (Hupkes et al. 2019b)
- Visualisation and ‘diagnostic classifiers’ reveal how recurrent and recursive neural networks process hierarchical structure (Hupkes, Veldhoen, and Zuidema 2018)
- Learning compositionally through attentive guidance (Hupkes et al. 2019a)
- Diagnostic classification and symbolic guidance to understand and improve recurrent neural networks (Hupkes and Zuidema 2017)



# Natural language

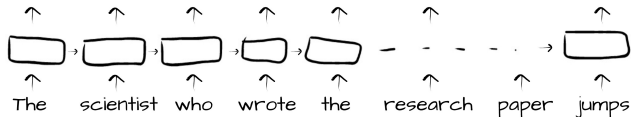
## Language modelling

The scientist who wrote the research paper ... ?

# Natural language

## Language modelling

The scientist who wrote the research paper ... ?



- Does such a model capture hierarchical structure?

## Subject-verb agreement

The **scientist** who wrote the research paper **jumps**

## Subject-verb agreement

The **scientist** who wrote the research paper **jumps**

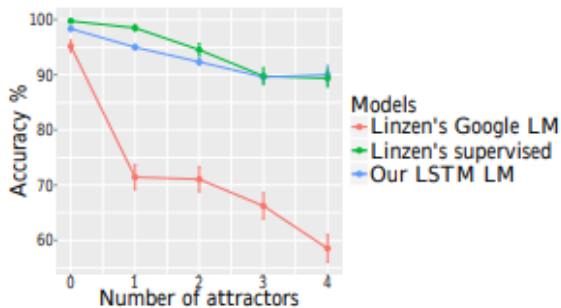
The **scientists** who wrote the research paper **jump**

## The number agreement task

The **scientists** who wrote the research paper ... **jump/ jumps?**

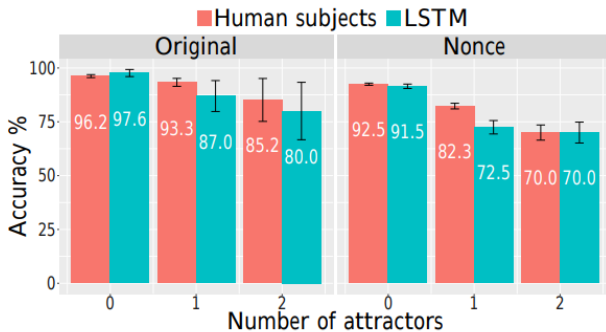
(Linzen, Dupoux, and Goldberg 2016)

## Results



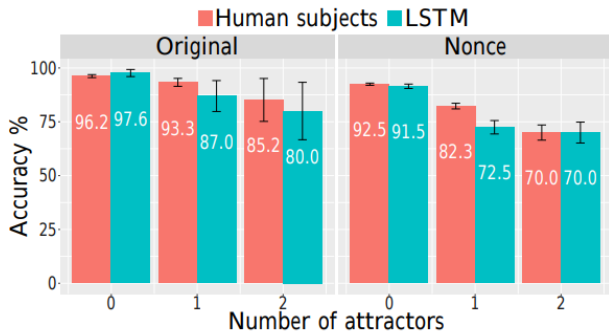
(Gulordava et al. 2018)

## Original and nonsensical sentences



(Gulordava et al. 2018)

## Original and nonsensical sentences

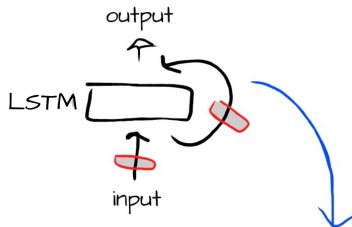


- But *how* do they do this?



## Diagnostic Classification

# Diagnostic Classification

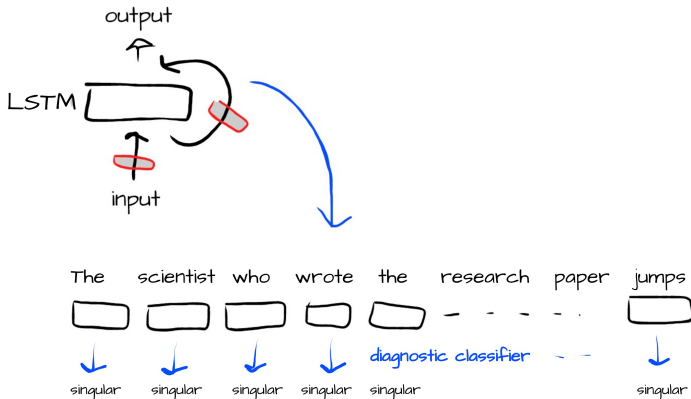


The scientist who wrote the research paper jumps

[ ] [ ] [ ] [ ] [ ] - - - [ ]

(Hupkes, Veldhoen, and Zuidema 2018; Veldhoen, Hupkes, and Zuidema 2016)

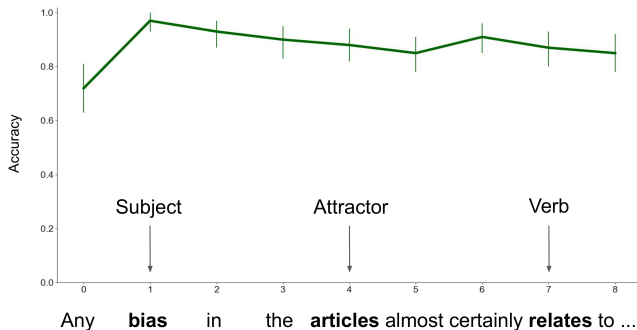
# Diagnostic Classification



(Hupkes, Veldhoen, and Zuidema 2018; Veldhoen, Hupkes, and Zuidema 2016)

# Diagnostic Classification

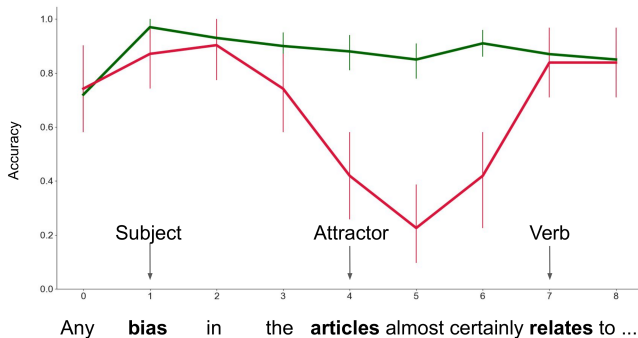
Sentences with correct predictions, h



(Giulianelli, Harding, Mohnert, Hupkes and Zuidema)

# Diagnostic Classification

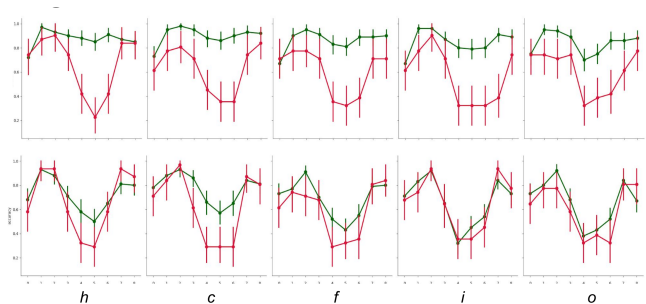
All sentences, h



(Giulianelli et al. 2018)

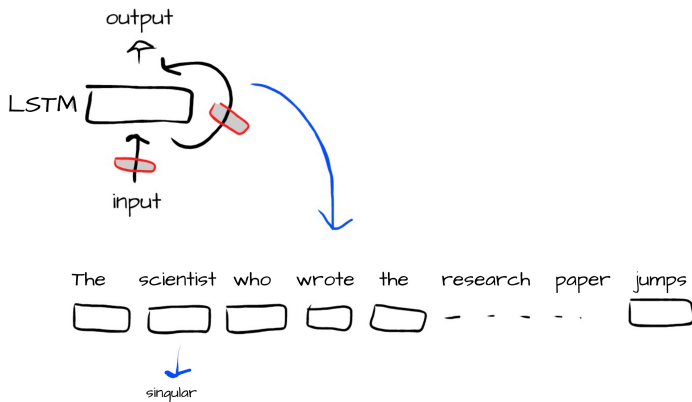
# Diagnostic Classification

All sentences, all components



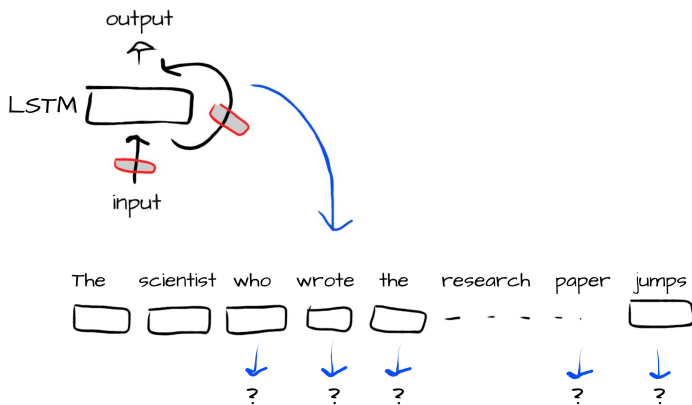
(Giulianelli et al. 2018)

# Temporal Generalisation



(Giulianelli et al. 2018)

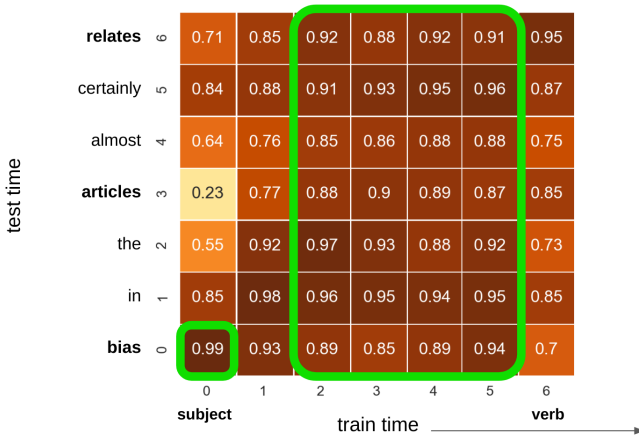
# Temporal Generalisation



(Giulianelli et al. 2018)

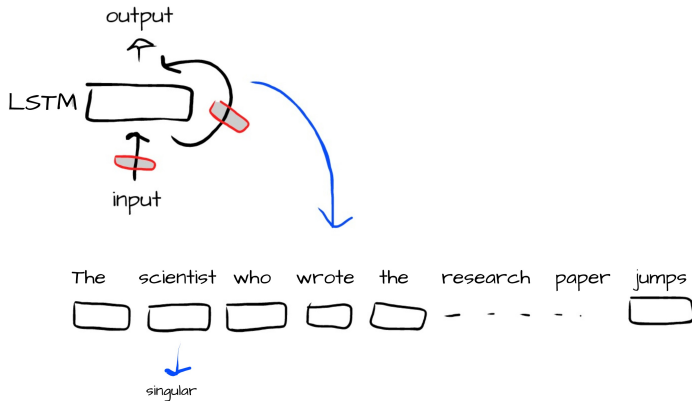


# Temporal generalisation matrix



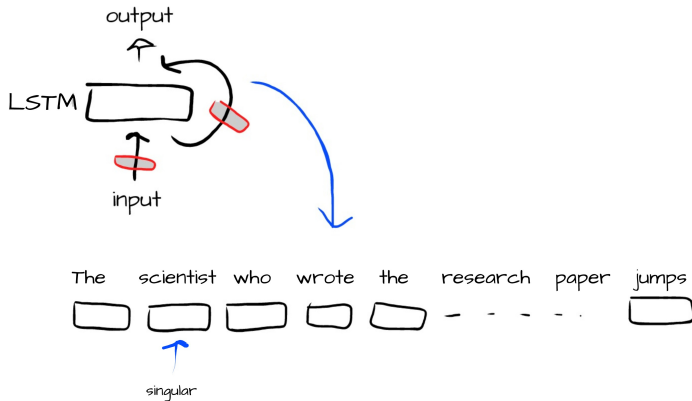
(Giulianelli et al. 2018)

# Diagnostic interventions



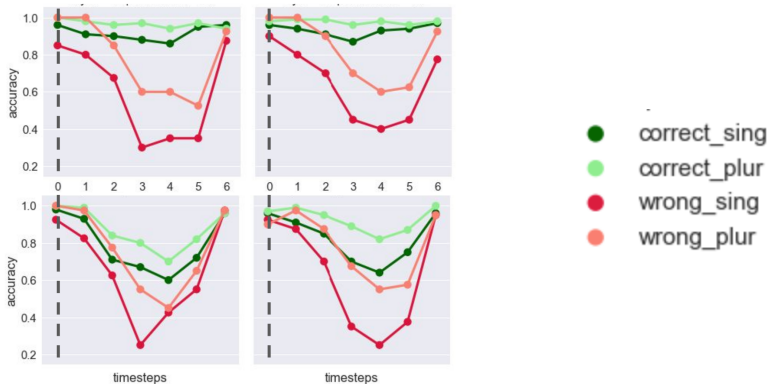
(Giulianelli et al. 2018)

# Diagnostic interventions



(Giulianelli et al. 2018)

# Diagnostic Interventions



(Giulianelli et al. 2018)

# Diagnostic interventions, results

	An	official	estimate	issued	in	2003	suggests	suggest
<b>Original</b>		-11.05	-8.426	-8.472	-1.243	-3.951	-5.753	<b>-5.6979</b>
<b>Intervention</b>		-11.05	-8.426	-8.472	-1.268	-3.97	<b>-5.691</b>	-6.4361



without intervention	with intervention
78.0	85.4

\* Overall differences in sentence perplexities are statistically insignificant

(Giulianelli et al. 2018)

## Conclusions

With *Diagnostic Classification* we can discover if, when and where information is represented in a recurrent neural network:

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- Number information is stored mostly in the hidden and cell states of the LSTM language model;
- The model maintains a *deep* and *surface* representation of number;
- The model is indeed distracted by the attractor, but for wrong trials, the encoding already goes wrong *before* the attractor;

# Conclusions

With *Diagnostic Classification* we can discover if, when and where information is represented in a recurrent neural network:

- Number information is stored mostly in the hidden and cell states of the LSTM language model;
- The model maintains a *deep* and *surface* representation of number;
- The model is indeed distracted by the attractor, but for wrong trials, the encoding already goes wrong *before* the attractor;
- We can influence the behaviour of the model by *inverting* the diagnostic classifiers.

## Ablation Studies

# Templates for number-agreement tasks

**Simple**

**Adv**

**2Adv**

**CoAdv**

**NamePP**

**NounPP**

**NounPPAdv**

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

## Templates for number-agreement tasks

**Simple**            the **boy greets** the guy

**Adv**

**2Adv**

**CoAdv**

**NamePP**

**NounPP**

**NounPPAdv**

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

## Templates for number-agreement tasks

<b>Simple</b>	the <b>boy greets</b> the guy
<b>Adv</b>	the <b>boy</b> probably <b>greets</b> the guy
<b>2Adv</b>	
<b>CoAdv</b>	
<b>NamePP</b>	
<b>NounPP</b>	
<b>NounPPAdv</b>	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

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<b>Simple</b>	the <b>boy greets</b> the guy
<b>Adv</b>	the <b>boy</b> probably <b>greets</b> the guy
<b>2Adv</b>	the <b>boy</b> most probably <b>greets</b> the guy
<b>CoAdv</b>	
<b>NamePP</b>	
<b>NounPP</b>	
<b>NounPPAdv</b>	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

## Templates for number-agreement tasks

<b>Simple</b>	the <b>boy greets</b> the guy
<b>Adv</b>	the <b>boy</b> probably <b>greet</b> s the guy
<b>2Adv</b>	the <b>boy</b> most probably <b>greet</b> s the guy
<b>CoAdv</b>	the <b>boy</b> openly and deliberately <b>greet</b> s the guy
<b>NamePP</b>	
<b>NounPP</b>	
<b>NounPPAdv</b>	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)



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<b>CoAdv</b>	the <b>boy</b> openly and deliberately <b>greets</b> the guy
<b>NamePP</b>	the <b>boy</b> near Pat <b>greets</b> the guy
<b>NounPP</b>	
<b>NounPPAdv</b>	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

# Templates for number-agreement tasks

<b>Simple</b>	the <b>boy greets</b> the guy
<b>Adv</b>	the <b>boy</b> probably <b>greets</b> the guy
<b>2Adv</b>	the <b>boy</b> most probably <b>greets</b> the guy
<b>CoAdv</b>	the <b>boy</b> openly and deliberately <b>greets</b> the guy
<b>NamePP</b>	the <b>boy</b> near Pat <b>greets</b> the guy
<b>NounPP</b>	the <b>boy</b> near the car <b>greets</b> the guy
<b>NounPPAdv</b>	

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

# Templates for number-agreement tasks

<b>Simple</b>	the <b>boy</b> <b>greet</b> s the guy
<b>Adv</b>	the <b>boy</b> probably <b>greet</b> s the guy
<b>2Adv</b>	the <b>boy</b> most probably <b>greet</b> s the guy
<b>CoAdv</b>	the <b>boy</b> openly and deliberately <b>greet</b> s the guy
<b>NamePP</b>	the <b>boy</b> near Pat <b>greet</b> s the guy
<b>NounPP</b>	the <b>boy</b> near the car <b>greet</b> s the guy
<b>NounPPAdv</b>	the <b>boy</b> near the car kindly <b>greet</b> s the guy

(Lakretz, Kruszewski, Desbordes, Hupkes, Dehaene and Baroni)

# Ablation Results

NA task	Condition	Full Model
Simple	S	100
Adv	S	100
2Adv	S	99.9
CoAdv	S	98.7
namePP	SS	99.3
nounPP	SS	99.2
nounPP	SP	87.2
nounPPAdv	SS	99.5
nounPPAdv	SP	91.2
Simple	P	100
Adv	P	99.6
2Adv	P	99.3
CoAdv	P	99.3
namePP	PS	68.9
nounPP	PS	92.0
nounPP	PP	99.0
nounPPAdv	PS	99.2
nounPPAdv	PP	99.8

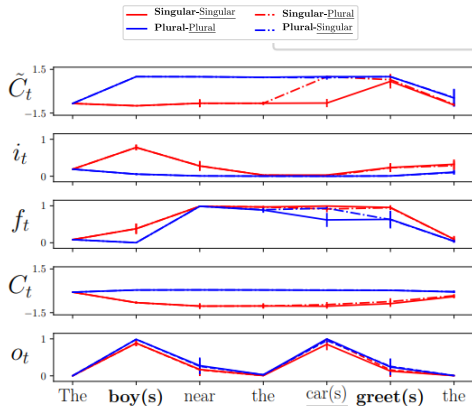
# Ablation Results

NA task	Condition	Full Model	Ablated	
			776	988
Simple	S	100	-	-
Adv	S	100	-	-
2Adv	S	99.9	-	-
CoAdv	S	98.7	-	82
namePP	SS	99.3	-	-
nounPP	SS	99.2	-	-
nounPP	SP	87.2	-	54.2
nounPPAdv	SS	99.5	-	-
nounPPAdv	SP	91.2	-	54.0
Simple	P	100	-	-
Adv	P	99.6	-	-
2Adv	P	99.3	-	-
CoAdv	P	99.3	79.2	-
namePP	PS	68.9	39.9	-
nounPP	PS	92.0	48.0	-
nounPP	PP	99.0	78.3	-
nounPPAdv	PS	99.2	63.7	-
nounPPAdv	PP	99.8	-	-

# Singular unit behaviour

$$c_t = f_t \circ c_{t-1} + i_t \circ \tilde{c}_t$$

$$h_t = o_t \circ \tanh(c_t)$$



(a) 988 (singular)

(Lakretz et al. 2019)

## Diagnostic Classification 2

- Short distance relations?

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- Short distance relations?
  - → Diagnostic classifiers to predict *number* information



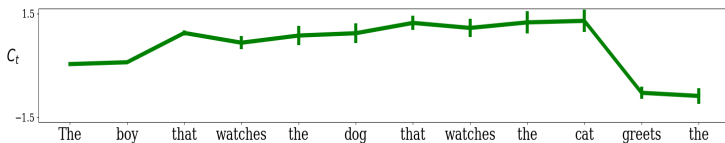
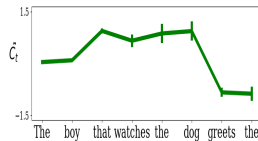
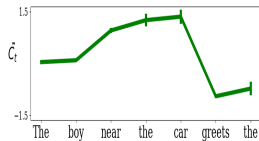
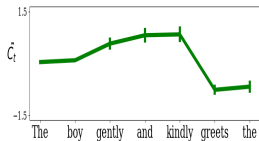
## Diagnostic Classification 2

- Short distance relations?
  - → Diagnostic classifiers to predict *number* information
- The syntactic structure?

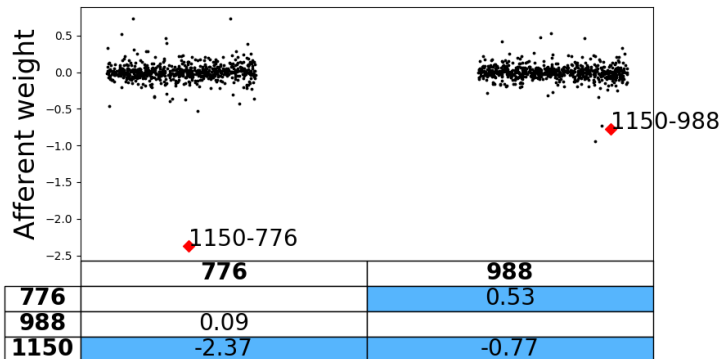
## Diagnostic Classification 2

- Short distance relations?
  - → Diagnostic classifiers to predict *number* information
- The syntactic structure?
  - → Diagnostic classifiers to predict *syntactic depth*

# Syntax unit 1150, cell activity



# Syntax unit 1150, outgoing weights

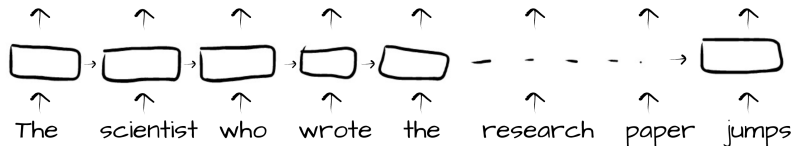


# Conclusions

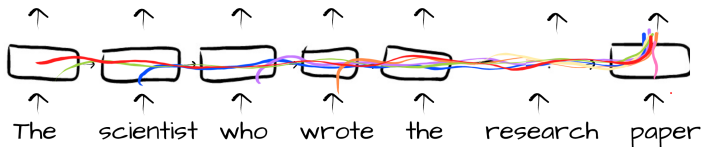
- Using **ablation**, we found that long distance number is encoded locally, in two units;
  - One *singular* unit
  - One *plural* unit
- Using **diagnostic classifiers and ablation**, we found that short distance number is encoded in a distributed fashion;
- Using **diagnostic classification**, we found a number of syntax units, one of which highly interpretable.

# Generalised Contextual Decomposition

# Contextual Decomposition



# Contextual Decomposition

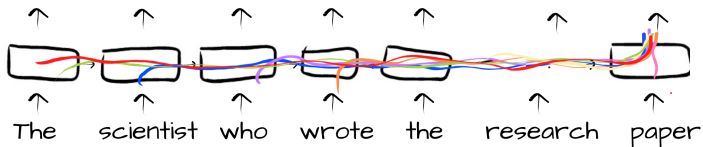


- Keep track of interactions

(Murdoch, Liu, and Yu 2018)

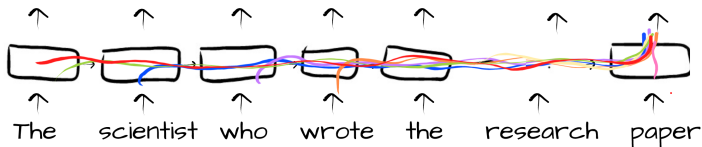


# Contextual Decomposition



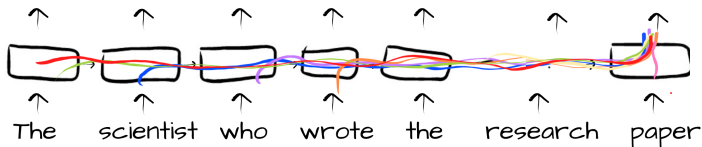
- Keep track of interactions
  - Linear sums:  $3 * 2 + 1 * 4$

# Contextual Decomposition



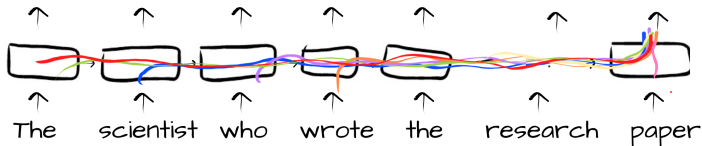
- Keep track of interactions
  - Linear sums:  $3 * 2 + 1 * 4$
  - Non-linearities:  $\text{TANH}(10 + 20)$

# Contextual Decomposition



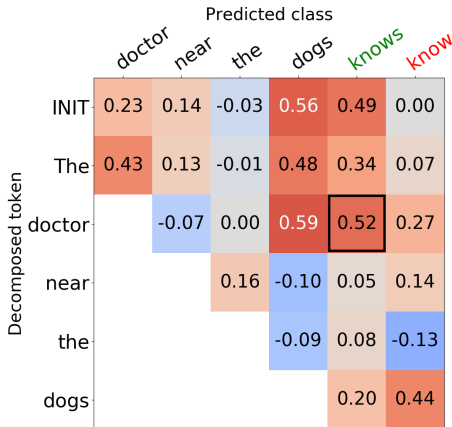
- Keep track of interactions
  - Linear sums:  $3 * 2 + 1 * 4$
  - Non-linearities:  $\text{TANH}(10 + 20)$
  - Multiplications:  $5 * 2$

# Contextual Decomposition



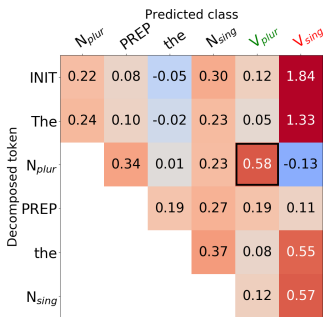
- Keep track of interactions
  - Linear sums:  $3 * 2 + 1 * 4$
  - Non-linearities:  $\text{TANH}(10 + 20)$
  - Multiplications:  $5 * 2$
- Which interactions?

# Information flow “attention” plots



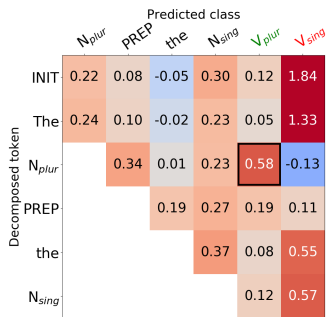
(Jumelet, Hupkes, and Zuidema 2019)

# Singular versus plural

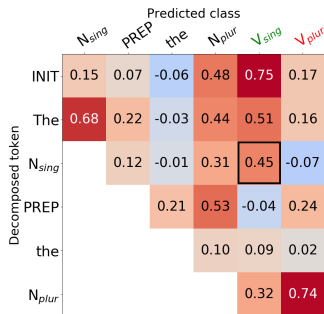


NounPP – PS

# Singular versus plural



NounPP – PS



NounPP – SP

# Pruning information

			GCD
Task	Condition	FULL	IN
Simple	S	100	73.3
Simple	P	100	100
nounPP	SS	99.2	93.0
nounPP	SP	87.2	90.3
nounPP	PS	92.0	100
nounPP	PP	99.0	100
namePP	SS	99.3	97.7
namePP	PS	68.9	98.3

- FULL: full model accuracy
- IN: information from the subject,



# Pruning information

Task	Condition	FULL	GCD	
			IN	INTERCEPT*
Simple	S	100	73.3	97.3
Simple	P	100	100	32.7
nounPP	SS	99.2	93.0	99.8
nounPP	SP	87.2	90.3	98.8
nounPP	PS	92.0	100	0.0
nounPP	PP	99.0	100	7.0
namePP	SS	99.3	97.7	99.4
namePP	PS	68.9	98.3	1.3

- FULL: full model accuracy
- IN: information from the subject,
- INTERCEPT\*: only intercept interactions

## Pruning information

Task	Condition	FULL	GCD		
			IN	INTERCEPT*	$\neg$ INTERCEPT
Simple	S	100	73.3	97.3	69.7
Simple	P	100	100	32.7	100
nounPP	SS	99.2	93.0	99.8	72.7
nounPP	SP	87.2	90.3	98.8	60.5
nounPP	PS	92.0	100	0.0	100
nounPP	PP	99.0	100	7.0	99.8
namePP	SS	99.3	97.7	99.4	76.2
namePP	PS	68.9	98.3	1.3	99.9

- FULL: full model accuracy
- IN: information from the subject,
- INTERCEPT\*: only intercept interactions
- $\neg$ INTERCEPT: no intercept interactions

# Conclusions

We can use contextual decomposition to track the information flow in recurrent neural networks:

- Plural verbs have a much stronger causal relationship to their plural subject than singular verbs to their singular subject.
- By considering different types of interactions, we find that to predict singular verbs, the model relies heavily on its intercepts
- GCD can also be used in other kinds of scenario's, where behavioural accuracy tests are not possible (anaphora resolution, negative polarity items)!

What's next?

# Thanks to my collaborators



Willem Zuidema



Marco Baroni



Jaap Jumelet



Germán Kruszewski



Yair Lakretz



Sara Veldhoen



Mario Giulianelli



Florian Mohnert



Jack Harding

# Special thanks



Willem Zuidema



Jaap Jumelet

## Special thanks



Willem Zuidema



Jaap Jumelet

<https://github.com/i-machine-think/diagnnose>

What's next?



# What's next?

- Other linguistic questions

# What's next?

- Other linguistic questions
  - Negative polarity items (Jumelet and Hupkes 2018; Marvin and Linzen 2018)
  - Filler-gap dependencies (Wilcox et al. 2018, 2019)
  - Reflexive anaphora (Futrell et al. 2019; Jumelet, Hupkes, and Zuidema 2019; Marvin and Linzen 2018)
  - Garden path sentences (Futrell et al. 2019; Van Schijndel and Linzen 2018; Wilcox et al. 2019)
  - Syntactic priming (Prasad, Schijndel, and Linzen 2019; Van Schijndel and Linzen 2018)
  - And many more. . .
- Other “model” questions

# What's next?

- Other linguistic questions
- Other “model” questions
  - Do structural biases help? (Futrell et al. 2018; Wilcox et al. 2019)
  - What is the impact of quantity and quality of training data (Schijndel, Mueller, and Linzen 2019)?

# What's next?

- Other linguistic questions
- Other “model” questions
- The ultimate question

# What's next?

- Other linguistic questions
- Other “model” questions
- The ultimate question
  - How does this help us to better understand human language processing?

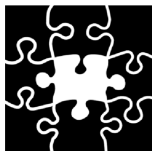
# What's next?

- Other linguistic questions
- Other “model” questions
- The ultimate question
  - How does this help us to better understand human language processing?

**I'm looking forward to figuring those things out!**

# Thank you

Thank you for your attention!



ILLC



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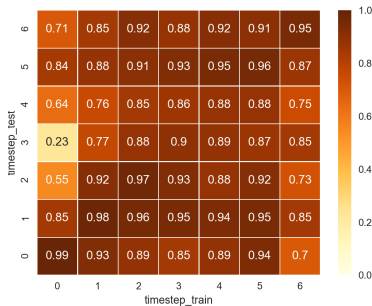
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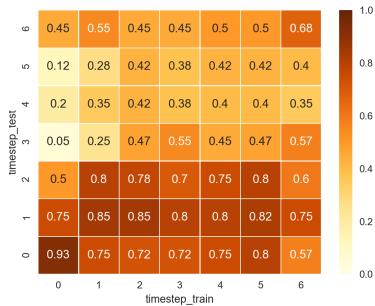
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## Appendices

# Temporal Generalisation



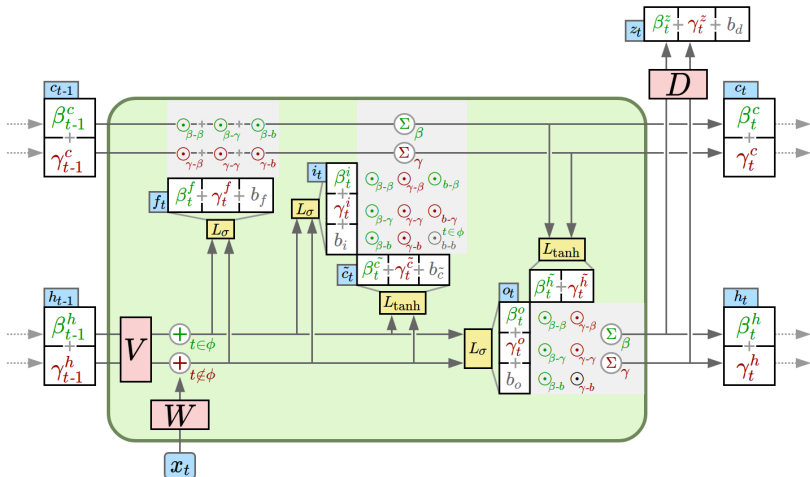
Correct trials



Wrong trials

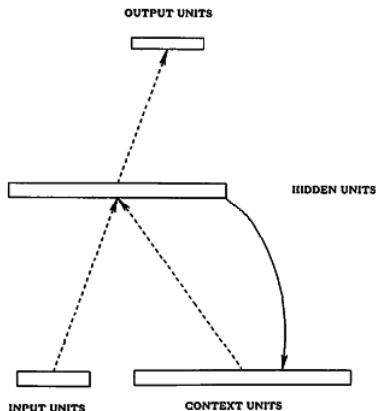


# Generalised Contextual Decomposition



# Simple Recurrent Network

$$\mathbf{h}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{b})$$

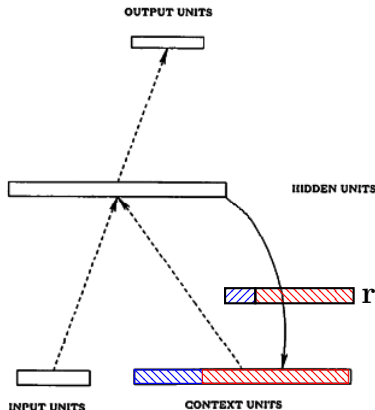


(Elman 1990)

# Gated recurrent neural networks

$$\mathbf{h}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}\mathbf{h}_{t-1} + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r\mathbf{x}_t + \mathbf{U}_r\mathbf{h}_{t-1} + \mathbf{b}_r)$$

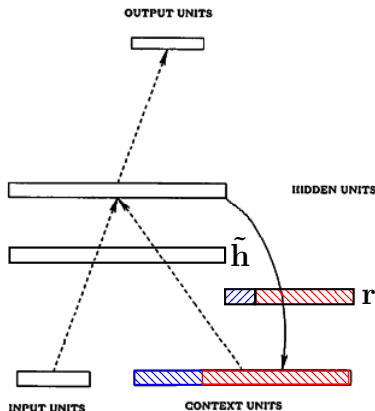


(Cho et al. 2014; Chung et al. 2015)

# Gated recurrent neural networks

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}(\mathbf{r} \odot \mathbf{h}_{t-1}) + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r\mathbf{x}_t + \mathbf{U}_r\mathbf{h}_{t-1} + \mathbf{b}_r)$$



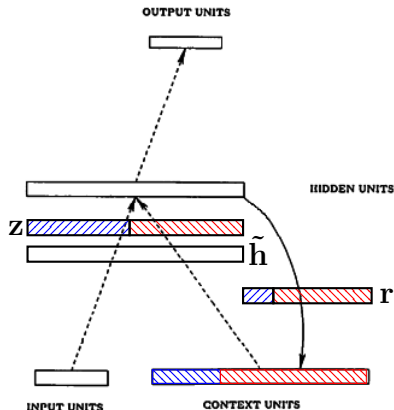
(Cho et al. 2014; Chung et al. 2015)

# Gated recurrent neural networks

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}(\mathbf{r} \odot \mathbf{h}_{t-1}) + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r\mathbf{x}_t + \mathbf{U}_r\mathbf{h}_{t-1} + \mathbf{b}_r)$$

$$\mathbf{z}_t = \sigma(\mathbf{W}_z\mathbf{x}_t + \mathbf{U}_z\mathbf{h}_{t-1} + \mathbf{b}_z)$$



(Cho et al. 2014; Chung et al. 2015)

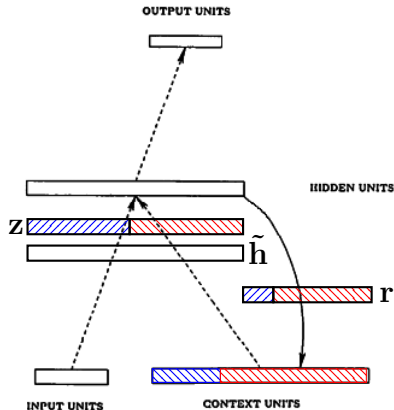
# Gated recurrent neural networks

$$\tilde{\mathbf{h}}_t = \tanh(\mathbf{W}\mathbf{x}_t + \mathbf{U}(\mathbf{r} \odot \mathbf{h}_{t-1}) + \mathbf{b})$$

$$\mathbf{r}_t = \sigma(\mathbf{W}_r\mathbf{x}_t + \mathbf{U}_r\mathbf{h}_{t-1} + \mathbf{b}_r)$$

$$\mathbf{z}_t = \sigma(\mathbf{W}_z\mathbf{x}_t + \mathbf{U}_z\mathbf{h}_{t-1} + \mathbf{b}_z)$$

$$\mathbf{h}_t = (1 - \mathbf{z}_t) \odot \mathbf{h}_{t-1} + \mathbf{z}_t \odot \tilde{\mathbf{h}}_t$$



(Cho et al. 2014; Chung et al. 2015)